Hedging performance of range-based volatility estimators

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Abstract

This study revisits the estimation of optimal hedge ratio by using range-based volatility estimators, which have lately evoked a good amount of interest for their efficiency. The optimal hedge ratios are estimated under the minimum variance framework, based on the weekly returns and range data from January 2000 to December 2016. Energy-related commodities (crude oil and natural gas), precious metals (gold and silver), and equity indexes (S&P 500 and Nifty 50) are examined for evaluating hedging effectiveness. The out of sample hedging performance is compared with the naïve hedge and various other return-based estimation methods. There is evidence that no specific econometric method outperforms the others consistently and significantly. The hedge ratios estimated with return-based estimators generally outperform those estimated with range-based estimators. Interestingly, the naïve hedge performs almost as well as the best performing strategy for all the asset classes.

*Keywords***:** Hedge ratio, range-based volatility estimators, commodity futures, minimum variance hedge ratio.

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Financial derivatives are widely used for hedging the underlying price exposure. Hedging can be seen as a special case of asset allocation for a two-asset portfolio, consisting of the underlying asset and its futures contract. One of the most important research questions in hedging strategies is the optimal hedge ratio, i.e., the number of futures contracts that are required to mitigate the spot risk. The optimal hedge ratio depends on the specific objective function to be optimized. Researchers have used different objective functions (minimum variance, mean-variance, expected utility, and semi-variance, to name a few) for estimating the hedge ratio.[1] Under certain conditions, the hedge ratios estimated from most of the objective functions will be equal to the 'minimum variance' hedge ratio (Chen, Lee, and Shrestha, 2003).

Hedgers try to mitigate the risk associated with the price changes in the underlying asset. Therefore, the success of a hedging strategy is measured by the extent to which it reduces the risk, that is, the variance in the price. In minimum variance strategies, the optimal hedge ratio is the ratio of covariance between the future and spot returns to the variance of the futures return. A number of econometric models have been employed to estimate the covariance between the spot and futures returns, and the variance of futures returns. Some early studies used ordinary least squares regression for this estimation (Ederington, 1979). With the advent of new econometric methods, more sophisticated models that account for heteroscedasticity, autocorrelation and cointegration were used (Baillie and Myers, 1991; Moschini and Myers, 2002; Park and Switzer, 1995). In the more recent studies, more complex methods, such as regime-switching and copula, are used (Hatemi-J & Roca, 2006; Alizadeh et al., 2008; Lee, 2010; Chang, 2011; Kostika & Markellos, 2013).

The covariance between futures and spot returns can also be calculated as the product of standard deviations of futures and spot returns, and the correlation between them. Thus, an accurate forecast of volatility plays an important role in estimating the hedge ratio. In this context, the range of daily prices has attracted special attention in recent years. For volatility estimation and forecasting, the range-based volatility estimators are claimed to be more efficient and to generate a more accurate volatility forecast (Vipul & Jacob, 2007; Hung et al., 2013; Molnár, 2012). In this study, we use various range-based volatility estimators for estimating the optimal hedge ratio and examine their performance relative to the traditional approaches like naïve and simple OLS methods. This study contributes to the existing literature in a number of ways. First, not much empirical work has been done on the hedging performance of range-based hedge ratios. We try to fill this gap by examining the performance of hedge ratios based on range-based volatility estimators. We compare their hedging performance with that of the traditional estimators and the return-based estimators. The range-based estimators include those based on the hybrid EWMA model, the CARR (conditional autoregressive range) model, and the Range-based GARCH model. Second, different asset classes like energy (crude oil and natural gas), precious metals (gold and silver), and equity indexes (S&P 500 and Nifty 50) are included for performance evaluation, to ensure robust results. Third, two performance evaluation metrics, variance, and semi-variance are used to examine the hedging effectiveness. As most of the return series are asymmetric, semi-variance provides a different dimension of risk, as it focuses on the downside of risk. To draw statistical inferences on the relative performance of different methods, Diebold & Mariano (1995) (DM) test and Mann-Whitney U test are used.

[1] For a detailed discussion on various objective functions and hedge ratios, please refer to Chen, Lee & Shrestha (2003)

The results indicate that the hedge ratios estimated from range-based methods provide no major advantage over the conventional methods. In fact, there is no significant difference between the best performing estimation model and the naïve hedge ratio. There is evidence that no single approach dominates in all the markets, and for all the performance measures.

The rest of the article is organized as follows. The next section provides a brief overview of the literature. The third section describes the data on the four commodity markets and two equity markets. The fourth section discusses the econometric models used for the estimation of optimal hedge ratios. The fifth section discusses the results, and the last section concludes the article.

Literature review

Many researchers have attempted to improve the estimation of the optimal hedge ratio, which is at the heart of hedging effectiveness. Johnson (1960) and Stein (1961) employed the concept of portfolio theory by hedging the spot position with futures. Early researchers used the classic linear regression of spot returns on futures returns for estimating the optimal hedge ratio (Ederington, 1979). The main disadvantage of OLS is that it ignores the autocorrelation, heteroscedasticity, and cointegration characteristics of a financial time series. Myers & Thompson (1989) pointed out that OLS regression leads to an error in Optimal Hedge Ratio (OHR) estimation. A generalized approach based on the conditional co-variance and conditional variance was proposed, which could provide better estimates than OLS. Chou et al. (1996) used the concept of cointegration between spot prices and futures prices and estimated the OHR using the error correction model. After the advent of GARCH models, specifically the multivariate GARCH model, many studies applied the GARCH framework to improve the hedging efficiency (in terms of reduction in variance). The conditional hedge ratio, based on the GARCH model, appeared to perform better than the unconditional OLS-based hedge ratio (Baillie & Myers, 1991; Park & Switzer, 1995; Moschini & Myers, 2002). Baillie & Myers (1991) estimated the OHR, using the bivariate GARCH model, and concluded that the OHR was time-varying. Park & Switzer (1995) examined the risk-minimizing futures hedge ratio of the stock index using the bivariate cointegrated model. Both the in-sample and out-of-sample performance of the dynamic hedging strategy appeared to be better than that of the constant hedging strategy. On the other hand, Lien et al. (2002) found that the hedge ratios based on the conditional hedging strategy do not provide any advantage over those based on the OLS strategy.

McMillan (2005) examined the performance of time-varying hedge ratios estimated from bivariate GARCH and GARCH-X, for the spot-futures portfolios of six non-ferrous metals, against the naïve and OLS hedge ratios. The results indicated that the hedge ratio estimated from the bivariate GARCH-X model provides the most effective hedge in five of the six cases. Ku et al. (2007) used the Dynamic Conditional Correlation (DCC) framework of the multivariate GARCH model in foreign exchange markets. The study compared DCC GARCH with CCC (Constant Conditional Correlation) GARCH, error correction model, and OLS based model to estimate the optimal hedge ratio. DCC model yielded the best result, while the traditional CCC GARCH model performed the worst. Lien & Yang (2008) evaluated different hedging strategies in Chinese metal futures contracts, using bivariate fractionally integrated generalized autoregressive conditional heteroskedasticity (BFIGARCH) model. They found the asymmetric BFIGARCH based optimal hedging strategy to provide the best hedging, for both in-sample and out-of-sample applications. Power et al. (2013) used a non-parametric Copula-based GARCH model to estimate the time-varying hedge ratios for live cattle and corn markets. They found the copula-based dynamic hedging to perform better than the static DCC GARCH and BEKK[2] GARCH models, in terms of the expected shortfall, with no significant reduction in portfolio variance. Kostika & Markellos (2013) studied the importance of forecasting higher moments in estimating the optimal hedge ratio, using ARCD (Auto regressive conditional density function). The performance is compared with GARCH-BEKK, EWMA, OLS, and error correction models. The ARCD approach had the best performance.

Alexander & Barbosa (2009) examined the performance of minimum variance hedging over naïve and simple OLS methods. They found that the minimum variance hedge ratios were more effective only for the markets that are less efficient. Moreover, in these cases, there was no significant improvement in the hedging performance of minimum variance hedge ratios, over the OLS-based hedge ratios. Wang et al. (2015) investigated the out of sample performance of 24 futures markets, using 18 econometric models based on minimum variance framework, and compared the performance with the naïve hedging strategy. They found that none of the 18 econometric models outperformed the naïve hedging strategy consistently and significantly. Therefore, the overall results are inconclusive, and it appears that more work needs to be done to ascertain the benefits of more sophisticated variance-covariance estimation methods for determining OHR for different markets.

Data

Weekly data has been employed for this study for the period from January 2000 to December 2016, except for NIFTY 50 (of National Stock Exchange of India) and natural gas. For Nifty 50, the data ranges from June 2000 to December 2016, because the futures trading on Nifty index started on 12th June 2000. For natural gas, the data is from April 2001 to December 2016, as the spot prices were not available before April 2001 in the Bloomberg database. Table 1 lists the futures contracts, their underlying spot markets, the data period, and the frequency. The data consists of six futures contracts on different assets, including crude oil, natural gas, gold, silver, S&P500, and NIFTY 50 indices. The return series have been calculated as the log difference between the closing prices of the current week and the previous week.

[2] BEKK is multivariate GARCH models of Baba, Engle, Kraft and Kroner

Table 1

Sample data period and source

Sector	Assets	Time Period	Spot market	Futures Exchange	Frequency
Energy	Crude Oil	07 Jan 2000 to 31 Dec 2016	West Texas intermediate, Cushing	NYMEX	Weekly
	Natural Gas	06 Apr 2001 to 31 Dec 2016	Henry Hub Natural Gas	NYMEX	Weekly
Bullion	Gold	07 Jan 2000 to 30 Dec 2016	Bloomberg Spot price	COMEX	Weekly
	Silver	07 Jan 2000 to 31 Dec 2016	Bloomberg Spot price	COMEX	Weekly
Equity	S&P 500	07 Jan 2000 to 31 Dec 2016	New York Stock Exchange	CME	Weekly
	NIFTY 50	18 Jun 2000 to 25 Dec 2016	National Stock Exchange of India (NSE)	NSE	Weekly

Table 2 presents the descriptive statistics of the sample spot and futures series. The mean return is positive for all the series, except for natural gas. NIFTY 50 has the highest mean return, followed by gold, and natural gas has the lowest return, followed by the S&P 500. The energy-related commodities are highly volatile, whereas gold is the least volatile.

Table 2

Descriptive statistics of spot and futures markets

Panel A: Futures markets						
	Crude Oil	Gold	Silver	Natural Gas	S&P 500	NSE
Mean	0.0940	0.1585	0.1269	-0.0450	0.0481	0.1960
Variance	22.5429	6.3159	19.1035	52.0425	6.3710	10.1395
Skewness	-0.5952	-0.2723	-1.3005	0.0291	-0.9430	-0.5514
Kurtosis	2.6659	1.6416	7.2187	1.0200	8.6712	3.0680
Maximum	21.4172	12.3461	14.6201	24.6512	12.2597	14.3237
Minimum	-26.2561	-10.1414	-31.9893	-28.5999	-21.8238	-18.2269
Jarque Bera	314.6827	110.4322	2,173.4762	35.6616	2,907.0976	381.7578
Panel B: Spot markets						
	Crude Oil	Gold	Silver	Natural Gas	S&P 500	NSE
Mean	0.0899	0.1590	0.1268	-0.0455	0.0497	0.1958
Variance	28.6650	6.0084	17.7859	87.2156	6.2312	9.8721
Skewness	-0.4320	-0.2088	-1.1309	0.2369	-0.8416	-0.5674
Kurtosis	5.2047	1.7774	5.9979	6.0252	7.1100	3.0701
Maximum	35.9391	13.2036	14.9149	52.5781	11.3559	14.3568
Minimum	-31.2180	-8.9477	-29.5998	-45.0729	-20.0837	-17.3755
Jarque Bera	1,027.6047	123.0650	1,516.9073	1,248.0370	1,970.8267	384.7970

Jarque Bera test statistics is significant at 1% for all the assets.

In addition, the volatility of the spot market is greater than that of its futures market for energy-related commodities. This may be attributed to the fact that the spot markets of energy commodities are less liquid than their futures market. All the spot and futures returns are negatively skewed, excepting those for natural gas, and are leptokurtic. The Jarque-Bera statistics confirm the non-normality for all the series.

Methodology

In this study, we use minimum variance as the objective function, as a hedger is more concerned about the risk rather than the return. Suppose a hedger is holding a commodity and wants to hedge its price risk by using futures of the underlying commodity. At time t, the hedger wants to know the number of futures contracts required for minimizing the variance of a portfolio, consisting of the underlying commodity and its corresponding futures contract.

The variance of the hedged portfolio would be

$$var\left[R_{h,t+1}\right] = var\left[R_{s,t+1} - h_t R_{f,t+1}\right] \tag{1}$$

where h_t is the hedge ratio estimated at time t; and R_h, R_s and R_f are the returns of the hedged portfolio, the spot price, and the futures price, respectively. The optimal hedge ratio h_t* that minimizes the variance is

$$h_t^{\ *} = \frac{cov\left[R_{s,t+1}, R_{f,t+1}\right]}{var\left[R_{f,t+1}\right]} \tag{2}$$

$$= \rho_{(R_{s,t+1}, R_{f,t+1})} \frac{\sigma_{R_{s,t+1}}}{\sigma_{R_{f,t+1}}} \tag{3}$$

where $\rho_{(R_{s,t+1}, R_{f,t+1})}$ is the forecasted correlation coefficient between the spot return and futures return at time t. Thus, the better the forecast of the variances at time t, the better is the hedge ratio. Many models based on return series have been used to forecast the joint dynamics of futures and spot price, and to estimate the optimal hedge ratio. In the recent literature, a lot of interest has been generated in the volatility estimation based on price range. The forecasting performance of range-based volatility estimators is found to be better than that of the returns based estimators. We examine the performance of hedge ratios estimated with range-based estimators and compare it with the performance of return-based estimators. Researchers have studied the estimation and forecasting performance of three major range-based estimators that do not require high-frequency data: the Parkinson (PK) estimator (Parkinson, 1980), the Garman and Klass (GK) estimator (Garman & Klass, 1980), and the Rogers and Satchell (RS) estimator (Rogers & Satchell, 1991). The RS estimator is claimed to be more efficient than the PK and GK estimators in the presence of drift (Rogers et al., 1994). In this study, the RS estimator is used as the proxy for volatility, except for natural gas. For natural gas, the PK estimator is used as the proxy for volatility because the open price is not available for the natural gas spot market before April 2006. The RS estimator of volatility is given by

$$\hat{\sigma}_{RS}^2 = \frac{1}{n}\sum_{t=1}^{n}[\,(H_t - C_t)(H_t - O_t) + (L_t - C_t)(L_t - O_t)] \tag{4}$$

where σ_{RS} is the RS volatility, H_t, L_t, O_t and C_t are the log high, low, open, and close prices at time t, and n is the number of periods used for the volatility estimation.

Forecasting models for range-based variance

Autoregressive model

The Autoregressive (AR) model is given as

$$\sigma_{t,j} = \omega_j + \sum_{i=1}^{p} \alpha_{ij}\sigma_{t-i,j} + \varepsilon_{t,j} \tag{5}$$

where $\sigma_{t,j}$ is the volatility forecast using the RS estimator of asset j for time t, ω_j and α_{ij} are the parameters.

Range-based GARCH model

The Range-based GARCH model (RGARCH) suggested by Molnár (2016) is a simple modification of univariate GARCH (1,1) model. In this model, volatility is forecast using the GARCH model, with range-based volatility as an exogenous variable in the variance equation. The RGARCH model is given by

$$r_t = \mu_0 + \varepsilon_t \tag{6}$$

$$\varepsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\widehat{\sigma_{RS,t-1}^2} + \beta\sigma_{t-1}^2 \tag{7}$$

where ε_{t-1} is the innovation in the returns for the day t-1, $\widehat{\sigma_{RS,t-1}^2}$ is the estimated volatility with the RS estimator, and σ_{t-1} is the volatility on the day t-1.

Conditional autoregressive range (CARR) model

R. Y. Chou (2005) proposed the CARR model to capture the dynamic evolution of volatility by using the high-low range of asset prices, as the measure of volatility. The CARR model of order (p,q) follows.

$$R_t = \lambda_t\varepsilon_t$$

$$\lambda_t = \omega + \sum_{i=1}^{q} \alpha_i R_{t-i} + \sum_{j=1}^{p} \beta_j\, \lambda_{t-j} \tag{8}$$

$$\varepsilon_t \mid I_{t-1} \sim f(.)$$

where $R_t = lnH_t - lnL_t$; λ_t is the conditional mean of the range, based on all the information up to time t-1; I_{t-1} is the information available up to and including time t-1; and the distribution of the disturbance term ε_t is assumed to have a density function f(.), with unit mean. Since ε_t is positively valued (given that both R_t and λ_t are positively valued), a natural choice for the distribution is the exponential distribution. R. Y. Chou (2005) also shows that the parameters ω, α_i and β_j must meet the following conditions, for the model to be stationary, and to ensure nonnegative range.

$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$, and ω, α_i and $\beta_j > 0$

Hybrid EWMA model

In this study, we employ the hybrid Exponentially Weighted Moving Average (EWMA) model proposed by Harris & Yilmaz (2010). The hybrid EWMA model is a combination of range-based and return-based approaches. The conditional covariance is specified as the product of the range-based conditional standard deviations, and the returns-based conditional correlation coefficient.

The standard multivariate EWMA model for conditional variance-covariance matrix is given by

$$\sigma_{i\,j,t} = \lambda\sigma_{i\,j,t-1} + (1-\lambda)r_{i,t-1}r_{j,t-1} \tag{9}$$

where $\sigma_{i\,j,t}$ is the forecast covariance between assets 1 and 2 (*i=1, j=2* in a two-asset situation), and λ is the (single) decay factor. The correlation between the two assets is estimated as

$$\rho_{ij,t} = \frac{\sigma_{ij,t}}{\sqrt{\sigma_{ii,t}}\sqrt{\sigma_{jj,t}}}$$

The conditional variances of the two assets are estimated by employing a range-based EWMA specification.

$$\sigma_{ij,t}^{hybrid} = \lambda_1\sigma_{ij,t-1}^{hybrid} + (1-\lambda_1)s'_{i,t-1} \qquad for\ i=j \tag{10}$$

$$\text{where } s'_{i,t} = \frac{1}{4ln2}\left(P_{i,t}^{H} - P_{i,t}^{L}\right)^2 + \left(P_{i,t}^{O} - P_{i,t-1}^{C}\right)^2$$

The conditional variance equation [Equation (10)] is a univariate EWMA model for the range-based variance, with a single decay factor λ_1. The estimator $s'_{i,t}$ provides an unbiased estimate of the daily close-to-close return variance, by combining the range-based estimate for the ‘open market’ period, and the return-based estimate for the ‘closed market’ period.

Now, the conditional covariance between the two assets is estimated as the product of the conditional standard deviations of the individual assets, and the conditional correlation coefficient between them.

$$\sigma_{ij,t}^{hybrid} = \rho_{ij,t}\sqrt{\sigma_{ii,t}^{hybrid}}\sqrt{\sigma_{jj,t}^{hybrid}} \qquad for\ i \neq j \tag{11}$$

Forecasting models for return-based variance

Ordinary least squares model

This is the conventional method and easy to estimate. In this method, a linear regression model is estimated, and the coefficient of the independent variable is the estimated optimal hedge ratio. The linear regression model is

$$r_{s,t} = \alpha + \beta r_{f,t} + \varepsilon_{s,t} \tag{12}$$

where $\varepsilon_{s,t}$ ~ N(0,1). One of the major criticisms of this method is that the estimated OHR is constant. To overcome this shortcoming, this study uses rolling OLS regression to estimate the optimal hedge ratio (explained in Section 4.5).

Constant conditional correlation GARCH (CCC) model

Bollerslev (1990) derived the CCC model with the assumption that the conditional correlation between variables is constant. In this model, the covariance is decomposed into the product of correlation and standard deviations. The temporal variations in the covariance are determined solely by the conditional variances. The variance-covariance matrix is given by

$$H_t = D_t^{1/2} \Gamma D_t^{1/2} \tag{13}$$

where D_t is the diagonal matrix of conditional variances, and Γ is the constant correlation matrix.

$$D_t = diag(H_t) = \begin{pmatrix} h_{11,t} & 0 \\ 0 & h_{22,t} \end{pmatrix} \tag{14}$$

and,

$$h_{iit} = \omega_{i0} + \omega_{i1}\epsilon_{i,t-1}^2 + \omega_{i2} h_{ii,t-1} \tag{15}$$

Here, ω_{i0}, ω_{i1}, and ω_{i2} are the estimated parameters. The conditional variance $h_{ii,t}$ is estimated using the standard univariate GARCH model. The main advantage of the CCC model is that it can be easily estimated. Furthermore, if the conditional variances in D_t are all positive definite, and Γ is also positive definite, then the conditional covariance matrix H_t is guaranteed to be positive definite, for all t.

Bivariate BEKK model

Engle and Kroner (1995) proposed a new parameterization of H_t to ensure its positivity. The covariance matrix in the BEKK model is given by

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}{}'A + B'H_{t-1}B \tag{16}$$

where *A, B,* and *C* are n x n matrices of parameters, and *C* is an upper triangular matrix. BEKK is expressed in the quadratic form to ensure that the H_t matrix is positive definite. The bivariate BEKK can be written as

$$\begin{aligned}\begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} \\ \sigma_{12,t} & \sigma_{22,t} \end{bmatrix} &= \begin{bmatrix} c_{11,t} & c_{21,t} \\ 0 & c_{22,t} \end{bmatrix} \begin{bmatrix} c_{11,t} & c_{21,t} \\ 0 & c_{22,t} \end{bmatrix}' \\ &+ \begin{bmatrix} a_{11,t} & a_{12,t} \\ a_{21,t} & a_{22,t} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-2} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-2} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11,t} & a_{12,t} \\ a_{21,t} & a_{22,t} \end{bmatrix}' \\ &+ \begin{bmatrix} b_{11,t} & b_{12,t} \\ b_{21,t} & b_{22,t} \end{bmatrix} \begin{bmatrix} \sigma_{11,t-1} & \sigma_{12,t-1} \\ \sigma_{21,t-1} & \sigma_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11,t} & b_{12,t} \\ b_{21,t} & b_{22,t} \end{bmatrix}' \end{aligned}$$

The parameters of the bivariate GARCH models are estimated by maximizing the log-likelihood function, assuming normally distributed errors. The log-likelihood function is given by

$$L(\theta) = -Tlog(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\log|H_t| + \varepsilon_t' H_t^{-1}\varepsilon_t)$$

where T is the total number of observations, and θ is the vector of parameters needed to be estimated. RATS software (version 9.10) is used for all the computations. Initial values of the parameters are obtained by simplex algorithms, and the final estimates of the parameters are obtained by using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm.

Estimation of hedge ratios

Except for the rolling OLS, multivariate GARCH models, and the hybrid EWMA model (Harris & Yilmaz, 2010), the rest of the econometric models used in this study estimate only the conditional volatility. For such models, the dynamic hedge ratio is estimated as

$$h_t^* = \rho_t \frac{\sigma_{R_{s,t+1}}}{\sigma_{R_{f,t+1}}}$$

where ρ_t is the rolling correlation between the spot and futures returns at time t, with a fixed window length of 500 weekly observations; and $\sigma_{R_{s,t+1}}$ and $\sigma_{R_{f,t+1}}$ are the t+1 day volatility forecasts of the spot and futures returns.

For multivariate GARCH models and hybrid EWMA model, the dynamic hedge ratio is estimated as the ratio of the covariance between the spot and futures returns to the variance of futures returns.

Hedging performance and inference

This study evaluates the out-of-sample hedging performance for six futures markets, using the rolling fixed window approach. The subsample of the first 500 weekly observations is used initially to estimate the parameters of the range-based and return-based models and to make the first forecast of the 'one-week ahead' optimal hedge ratio. For the next estimation, the first weekly observation from the estimation sample is dropped, and the first weekly observation from the remaining observations is added. This process is followed recursively to ensure that the total number of observations in each estimation sample remains 500. The model parameters are re-estimated every time the window is rolled forward, and new one-week ahead optimal hedge ratios are estimated. The hedging performance for all the methods is then compared with that of the best hedging strategy, i.e., the strategy for which the reduction in variance for the hedged portfolio is the maximum as compared to the unhedged portfolio.

In this study, two different performance measures are used: the percent reduction in variance (in the hedged portfolio, as compared to the unhedged portfolio), and the percent reduction in semi-variance. The variance of the hedged portfolio is var ($r_{s,t+1}$ - h_t $r_{f,t+1}$), where $r_{s,t+1}$ is the spot return at time t+1, $r_{f,t+1}$ is the futures return at time t+1, and h_t is the forecasted optimal hedge ratio for time t+1, estimated at time t. The hedging performance is evaluated by calculating

$$\text{Hedging Effectiveness (Variance)} = 1 - \left[\frac{Variance\ of\ hedged\ portfolio}{Variance\ of\ unhedged\ portfolio}\right]$$

To check whether the performance of the various methods differs significantly, we use the test of (Diebold & Mariano, 1995) (DM). In the DM test, let ε_i and ε_j are the residuals of two forecast at time t and let $d_i = \varepsilon_i^2 - \varepsilon_j^2$ is defined as the loss differential. The null hypothesis H_0 is that the expected losses from both of the residuals are equal, i.e., $E(d_i) = 0$. The DM test statistics is given by

$$DM = \frac{\bar{d}_i}{\hat{\sigma}_{\bar{d}_i}}$$

Where $\bar{d}_i = \frac{1}{n}\sum_{i=1}^{n} d_i$ and $\hat{\sigma}_{\bar{d}_i}$ is the consistent estimate of the standard deviation of $\bar{d}_i$. Under the null hypothesis, the test statistic DM follows a standard normal distribution. In our study, the loss at time t is defined by the variance of the hedged portfolio. If $f_{n,g} = \left(r_{s,t+1} - h_{t,g}\, r_{f,t+1}\right)^2$ is the loss at time t+1 associated with a strategy 'g' for which $h_{t,g}$ is the one step ahead forecast of the hedge ratio at time t. Then the loss differential at time t+1 is given by $d_i = (f_{n,g1} - f_{n,g2})$. If the DM test statistic < Z, where Z is the two-tailed critical value for the standard normal distribution, then the performance of the two models are same.

Semi-variance measures the dispersion of the observations that fall below the target return. It measures the downside risk, and is calculated as

$$\text{Semi-variance} = E\ \{(\max[0, \tau\text{-}R])^2\}$$

where τ is the target return, and R is the return of the hedged portfolio. The 4-week US Treasury Bill weekly rates from the Federal Reserve Economic Data (FRED) are used as the target return for this study. Since most of the return series are asymmetric, semi-variance may have additional useful information, as it focuses on the downside of risk. The hedging performance is measured in a similar fashion with semi-variance, as with the variance.

$$\text{Hedging effectiveness (semi-variance)} = 1 - \left[\frac{Semi\ variance\ of\ hedged\ portfolio}{Semi\ variance\ of\ unhedged\ portfolio}\right]$$

A higher value of hedging effectiveness indicates better hedging, in terms of variance reduction or downside risk. To check whether the performance of the various methods differs significantly, Wilcoxon-Mann-Whitney test is used. The power-efficiencies of the test are more than 95% as compared to the t-test if the assumptions of t-test are met (Siegel & Castellan (1988)).

Results

The hedging performance of various strategies is reported in tables 3 and 4. Table 3 presents the variance of hedged portfolio returns based on different strategies. Table 4 lists the percentage variance reduction due to hedging, as compared to the unhedged portfolio. All the hedging strategies have reduced the variance by more than 90% for all the assets, except for natural gas. For equity and precious metals, there is not much difference in the variance reduction, which is different from the behaviour of energy products. This can be attributed to the fact that the energy spot markets are less liquid and much more volatile than the other markets. Therefore, the hedging effectiveness becomes more sensitive to the strategy adopted. The strategy that causes the highest reduction in variance (or has the lowest variance) is the best strategy, and is highlighted in bold fonts in tables 3 and 4. Out of six markets, BEKK GARCH and rolling OLS account for the lowest variance in two markets each. CCC GARCH and naïve strategy cause the lowest variance in one market each. None of the range-based models has the lowest variance, but the hybrid EWMA model is the second-best strategy in the silver market. Among the range-based models, CARR and hybrid EWMA are the best, as they account for the lowest variance in three markets each. For crude oil and gold, the best strategy is BEKK GARCH, with 97.27% and 97.47% reduction in variance respectively, whereas hybrid EWMA provides a 97.23% reduction in variance in the gold market.

Table 3

Out of sample return variance of various strategies

	Unhedged	Naïve	Rolling OLS	AR Range	Hybrid EWMA	BEKK GARCH	CCC GARCH	Range GARCH	CARR
Crude Oil	28.6650	0.8177	0.8347	2.5601	1.1820	**0.7818**	0.8645	1.9927	1.9350
Gold	6.0084	0.1731	0.1616	0.2104	0.1664	**0.1522**	0.1750	0.2770	0.1965
Silver	17.7859	0.5931	**0.5394**	0.7036	0.5838	0.6870	0.5996	1.6272	0.6299
Natural Gas	87.2156	**23.4867**	23.7047	26.3987	26.2121	25.9652	25.9605	26.6692	25.2363
S&P 500	6.2312	0.0534	**0.0526**	0.0949	0.0728	0.0566	0.0532	0.0748	0.0719
NSE	5.2071	0.0623	0.0618	0.1250	0.1203	0.0629	**0.0615**	0.8019	0.0849

Values given in this table are the variance of the hedging models. The best performing model is highlighted in bold.

To test whether there is a significant difference between the best performing model and the remaining models, the DM test is employed. The best performing strategy is compared with the other strategies, one by one, for a particular commodity/index. The null hypothesis is that the two strategies selected for comparison are equally effective. The results are presented in Table 4, with *p*-values of DM test in parentheses. The test results indicate that, for BEKK, rolling OLS, and naïve strategies, the null hypothesis could not be rejected for any of the markets, at 5% level of significance. Therefore, all these strategies appear to be almost equally good if the objective is variance reduction. For CCC GARCH, the null hypothesis can be rejected for crude oil and silver, but not for the others. Among the range-based strategies, the null hypothesis could not be rejected in two markets for all the strategies. Overall, the range-based strategies are unable to outperform the GARCH based strategies and the naïve strategy.

Table 4

Out-of-sample hedging performance in terms of variance reduction

	Naïve	AR Range	Range GARCH	CARR	Hybrid EWMA	Rolling OLS	CC GARCH	BEKK
Crude Oil	97.15%	91.07%	93.05%	93.25%	95.88%	97.09%	96.98%	**97.27%**
	(0.454)	(0.00)	(0.00)	(0.00)	(0.00)	(0.19)	(0.04)	
Gold	97.12%	96.50%	95.39%	96.73%	97.23%	97.31%	97.09%	**97.47%**
	(0.09)	(0.07)	(0.03)	(0.00)	(0.26)	(0.33)	(0.23)	
Silver	96.67%	96.04%	90.85%	96.46%	96.72%	**96.97%**	96.63%	96.14%
	(0.05)	(0.02)	(0.18)	(0.01)	(0.04)		(0.01)	(0.31)
Natural Gas	**73.07%**	69.73%	69.42%	71.06%	69.95%	72.82%	70.23%	70.23%
		(0.08)	(0.12)	(0.25)	(0.22)	(0.86)	(0.06)	(0.24)
S&P 500	99.14%	98.48%	98.80%	98.85%	98.83%	**99.16%**	99.15%	99.09%
	(0.523)	(0.00)	(0.00)	(0.00)	(0.00)		(0.74)	(0.14)
NSE	98.80%	97.60%	84.60%	98.37%	97.69%	98.81%	**98.82%**	98.79%
	(0.712)	(0.00)	(0.00)	(0.17)	(0.00)	(0.85)		(0.28)

Values given are the percentage reduction in the return variance of the hedged position, as compared to that of the unhedged position. The values in parentheses are the p-values of DM test. The best performing model is highlighted in bold. To test the hypothesis of equal performance of all the hedging models against the best performing model, DM test is used.

Table 5 presents the semi-variance of hedged portfolio returns, based on different strategies. Table 6 shows the percentage reduction in semi-variance due to hedging, as compared to the unhedged portfolio. Out of the six markets, the rolling OLS produces the best results in three markets, BEKK GARCH in two markets, and CCC GARCH in one market. Among the range-based models, hybrid EWMA and CARR models account for the lowest variance, as compared to the other range-based models. To draw the statistical inferences, the Mann-Whitney U test is employed. The *p*-values are reported in Table 6 in parentheses. The results indicate that the null hypothesis, that there is no significant difference between the best and the tested strategy, cannot be rejected at 5% level of significance, for naïve, rolling OLS, RGARCH, and CCC GARCH models, for any of the markets. Therefore, all these strategies appear to be almost equally good, if the objective is semi-variance reduction. For BEKK GARCH, the null hypothesis is rejected only once. For CARR and hybrid EWMA models, it is rejected in two out of the six markets. The AR model performs the worst, as the null hypothesis is rejected for three out of the six markets.

Table 5

Out-of-sample semi-variance of the returns of various strategies

	Unhedged	Naïve	AR Range	Range GARCH	CARR	Hybrid EWMA	Rolling OLS	CC GARCH	BEKK
Crude Oil	0.10420%	0.00406%	0.01395%	0.00731%	0.01026%	0.00600%	0.00432%	0.00433%	**0.00396%**
Gold	0.02664%	0.00093%	0.00107%	0.00146%	0.00101%	0.00090%	0.00088%	0.00093%	**0.00081%**
Silver	0.10535%	0.00298%	0.00343%	0.01192%	0.00313%	0.00303%	**0.00282%**	0.00294%	0.00430%
Natural Gas	0.23761%	0.10872%	0.12522%	0.11902%	0.11496%	0.12219%	**0.10411%**	0.11624%	0.10970%
S&P 500	0.01937%	0.00023%	0.00043%	0.00022%	0.00031%	0.00030%	0.00021%	**0.00020%**	0.00024%
NSE	0.02367%	0.00036%	0.00047%	0.00381%	0.00039%	0.00061%	**0.00035%**	0.00035%	0.00035%

Values given in this table are % semi-variance of the returns of hedging models. The best performing model is highlighted in bold. For computing the semi-variance, 4-week Treasury Bill weekly rate has been used as the target return.

Table 6

Out-of-sample hedging performance in terms of semi-variance reduction

	Naïve	AR Range	Range GARCH	CARR	Hybrid EWMA	Rolling OLS	CC GARCH	BEKK
Crude Oil	96.10%	86.61%	92.98%	90.15%	94.24%	95.86%	95.84%	**96.20%**
	(0.27)	(0.00)	(0.08)	(0.00)	(0.00)	(0.82)	(0.26)	
Gold	96.51%	95.99%	94.53%	96.22%	96.61%	96.68%	96.52%	**96.95%**
	(0.83)	(0.30)	(0.87)	(0.19)	(0.74)	(0.83)	(0.33)	
Silver	97.17%	96.74%	88.68%	97.03%	97.13%	**97.32%**	97.21%	95.92%
	(0.39)	(0.06)	(0.33)	(0.20)	(0.46)		(0.80)	(0.77)
Natural Gas	54.24%	47.30%	49.91%	51.62%	48.58%	**56.19%**	51.08%	53.83%
	(0.65)	(0.89)	(0.30)	(0.88)	(0.49)		(0.49)	(0.29)
S&P 500	98.83%	97.78%	98.84%	98.40%	98.45%	98.89%	**98.99%**	98.76%
	(0.43)	(0.02)	(0.17)	(0.00)	(0.00)	(0.29)		(0.03)
NSE	98.49%	98.02%	83.91%	98.33%	97.44%	**98.54%**	98.53%	98.52%
	(0.61)	(0.97)	(0.58)	(0.54)	(0.23)		(0.85)	(0.47)

Values given are the percentage reduction in the semi-variance from the hedged model, as compared to the unhedged position, and the values in parentheses are the p-values of Mann-Whitney U test. The best performing model is highlighted in bold. To test the hypothesis of equal performance of all the hedging models against the best performing model, Mann-Whitney U test is used.

Overall, no single strategy is the best for all the markets, for all the performance measures. Further, there is no significant advantage of using the range-based models over the return-based models and the naïve model. The most surprising result is the performance of the naïve strategy, which performs the best for natural gas, with variance as the performance measure. More importantly, it is not significantly different from the best performing strategy for any commodity/index, with any of the performance measures (variance or semi-variance). Rolling OLS also performs reasonably well for both the performance measures, and particularly for the semi-variance. It indicates that the more sophisticated methods of forecasting variance and co-variance do not contain significant information for better hedging.

It is probable that the elaborate statistical procedures involved in these methods pick up noise, rather than the real information. Considering the challenges of implementation and only a marginal improvement in performance, the simple methods like naïve hedging and rolling OLS are clearly preferable to the more sophisticated methods, for identifying the optimal hedge ratio.

These results are in line with certain previous studies. Cotter & Hanly (2006) reported that the OLS and naïve strategies outperform the other strategies, for both the long and short hedgers, and for different performance measures. Alexander & Barbosa (2009) concluded that the minimum variance hedge ratios do not provide any benefit over the naïve and OLS strategies. Similarly, Wang et al. (2015) suggested that no other strategy outperformed the naïve strategy significantly and consistently, under the minimum-variance hedging.

Conclusion

Futures are important instruments in hedging. In this context, one of the most important research questions is the estimation of the optimal hedge ratio. A number of studies have examined the performance of the optimal hedge ratio estimated with various econometric methods, but there is no consensus on it. This study revisits the estimation of optimal hedge ratio by employing the range-based volatility estimators, which have attracted a good amount of interest in the recent literature for their higher efficiency.

The study is based on weekly data from January 2000 to December 2016 for six markets, including crude oil, gold, silver, natural gas, S&P 500, and NIFTY 50. The performance of hedge ratios estimated with various methods is evaluated with variance and semi-variance as the measures. The empirical results indicate that there is no specific econometric method, which outperforms the others consistently and significantly. The rolling OLS and BEKK GARCH methods are more efficient for estimating the optimal hedge ratio as compared to the other methods. The hedge ratios estimated with range-based models are not as efficient as those estimated with returns-based models are. More importantly, there is no significant difference in the variance/semi-variance of the naïve or rolling OLS based portfolios when compared to the other best-performing portfolios. The results are in line with some previous studies, which suggested that the simple naïve and OLS methods perform as well as the other more complex and sophisticated methods, if not better than them. The range-based methods are no exception to these general findings.

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